Latent Factor Models for Web Recommender Systems

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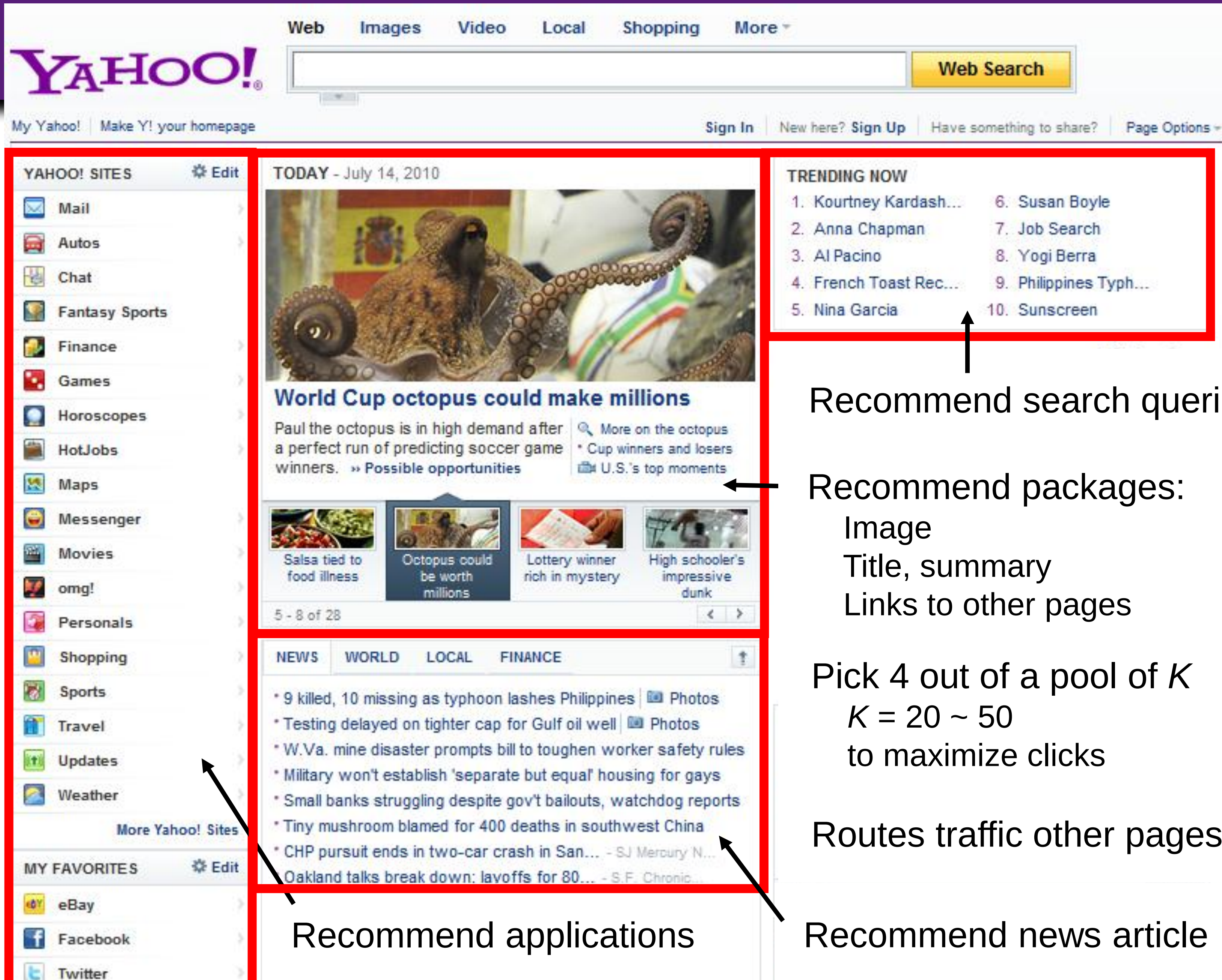
Outline

- Overview of recommender problems at Yahoo!
- Basics of matrix factorization
- Matrix factorization + feature-based regression
- Matrix factorization + topic modeling
- Matrix factorization + fast online learning
- Research problems beyond factor models
  - Explore/exploit (bandit problems)
  - Offline evaluation
  - Multi-objective optimization
  - Whole-page optimization

Web Recommender Systems

Recommend **items** to **users** to maximize some **objective(s)**



Recommend search queries

Recommend packages:
- Image
- Title, summary
- Links to other pages

Pick 4 out of a pool of K
- K = 20 ~ 50
- to maximize clicks

Routes traffic other pages

Recommend applications

Recommend news article

Web Recommender Systems

- Goal
  - Recommend **items** to **users** to maximize some **objective(s)**
- A new scientific discipline that involves
  - Machine Learning & Statistics (for learning user-item affinity)
    - Offline Learning
    - Online Learning
    - Collaborative Filtering
    - Explore/Exploit (bandit problems)
  - Multi-Objective Optimization
    - Click-rates (CTR), time-spent, revenue
  - User Understanding
    - User profile construction
  - Content Understanding
    - Topics, “aboutness”, entities, follow-up of something, breaking news,…

Recommend packages:
- Image
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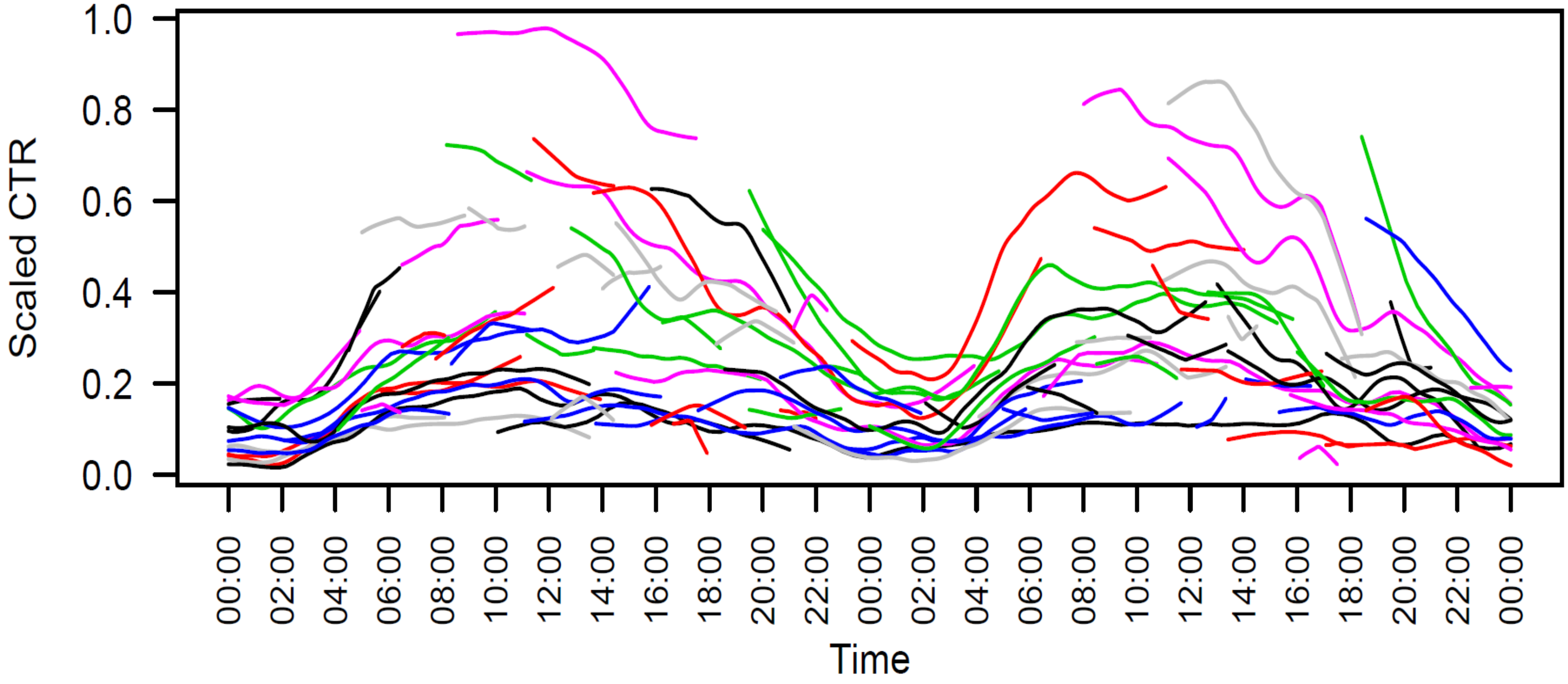
CTR Curves for Two Days on Yahoo! Front Page

Each curve is the CTR of an item in the Today Module on www.yahoo.com over time



Traffic obtained from a controlled randomized experiment (no confounding)

Things to note:

(a) Short lifetimes, (b) temporal effects, (c) often breaking news stories

Problem Definition

Algorithm selects item j with item features $\boldsymbol{x}_j$

(keywords, content categories, ...)

User i visits with user features $\boldsymbol{x}_i$

(demographics, browse history, geo-location, search history, …)

(i, j) : response y_{ij}

(click/no-click)

Which item should we select?

- The one with highest predicted CTR **Exploit**
- The one most useful for improving the CTR prediction model **Explore**

Our Strategy

	Most Popular Recommendation	Personalized Recommendation
Offline Learning		Collaborative filtering profile construction [KDD'09, WSDM'10]
Online Learning	Time-series models [WWW'09]	Online regression [NIPS'08]
Intelligent Initialization	Prior estimation	Prior estimation, dimension reduction [KDD'10]
Explore/Exploit	Multi-armed bandits [ICDM'09]	Bandits with covariates [Li, WWW'10]

Model Choices

- Feature-based (or content-based) approach
  - Use features to predict response
    - User features: Age, gender, geo-location, visit pattern, …
    - Item features: Category, keywords, topics, entities, …
    - Linear regression, Bayes Net, SVM, tree/forest methods, mixture models, …
  - Bottleneck: Need predictive features
    - Difficult to capture signals at granular levels: Cannot distinguish between users/items having same feature vectors
- Collaborative filtering (CF)
  - Make recommendation based on past user-item interaction
    - User-user, item-item, matrix factorization, …
    - See [Adomavicius & Tuzhilin, TKDE, 2005], [Konstan, SIGMOD'08 Tutorial]
  - Good performance for users and items with enough data
  - Does not naturally handle new users and new items (**cold-start**)

Factorization Methods

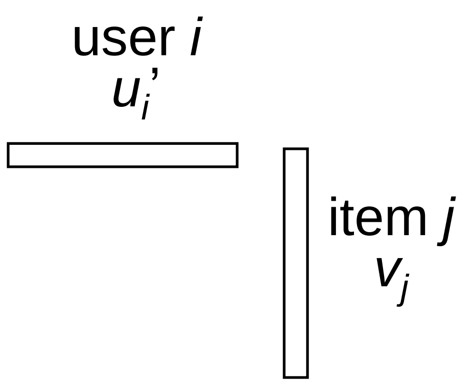
- Matrix factorization
  - Model each user/item as a vector of **factors** (learned from data)

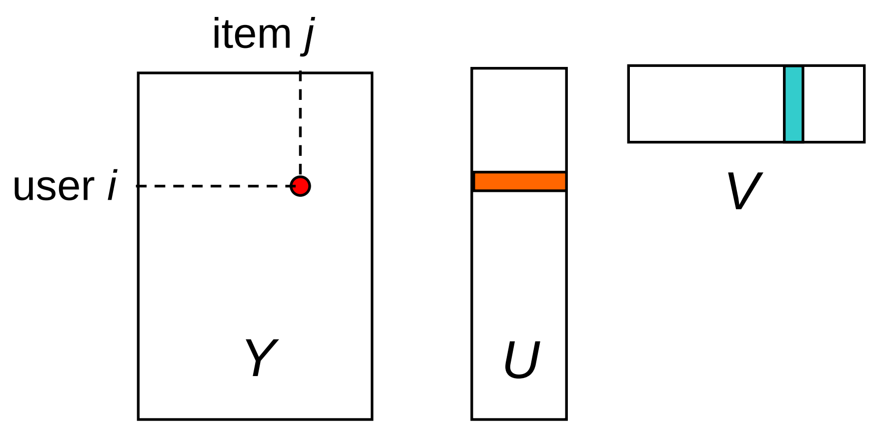
rating that user *i* gives item *j*

K << *M*, *N*
M = number of users
N = number of items

$$y_{ij} \sim \sum_k u_{ik} v_{jk} = u_i' v_j \quad \Leftrightarrow \quad \underset{M \times N}{Y} \sim \underset{M \times K}{U} \; \underset{K \times N}{V}$$

factor vector of user *i*   factor vector of item *j*





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factor vector of user *i* factor vector of item *j*

  – Better performance than similarity-based methods [Koren, 2009]
  – No factor for new items/users, and expensive to rebuild the model!!

- How to prevent overfitting
- How to handle cold-start
  – Use **features** (given) to predict the **factor values**

How to Prevent Overfitting

- Loss minimization

$$\ell(\mathbf{u},\mathbf{v}) =$$

$$\frac{1}{2\sigma^2}\sum_{(i,j)}(y_{ij} - u_i'v_j)^2$$

$$+\frac{1}{2\sigma_u^2}\sum_i \| u_i \|^2$$

$$+\frac{1}{2\sigma_v^2}\sum_j \| v_j \|^2$$

- Probabilistic model

$$y_{ij} \sim N(u_i'v_j,\ \sigma^2)$$

$$u_i \sim N(0,\ \sigma_u^2 I)$$

$$v_j \sim N(0,\ \sigma_v^2 I)$$

Given σ^2, σ_u^2, σ_v^2, find

$$\arg\min_{\mathbf{u},\mathbf{v}} \ell(\mathbf{u},\mathbf{v}) \xleftrightarrow{\text{equivalent}} \arg\max_{\mathbf{u},\mathbf{v}} \Pr[\mathbf{u},\mathbf{v} \mid \mathbf{y}]$$

How to set σ^2, σ_u^2, σ_v^2 ?

Probabilistic Matrix Factorization

- Probabilistic model

$$y_{ij} \sim N(u_i' v_j,\ \sigma^2)$$

$$u_i \sim N(0,\ \sigma_i^2 I)$$

$$v_j \sim N(0,\ \sigma_j^2 I)$$

$$\text{Let } \Theta = (\sigma^2, \sigma_u^2, \sigma_v^2)$$

$$\begin{aligned}\log \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta) &= \text{constant} \\ &- \tfrac{1}{2\sigma^2} \sum_{(i,j)} (y_{ij} - u_i' v_j)^2 - R \log \sigma^2 \\ &- \tfrac{1}{2\sigma_u^2} \sum_i \| u_i \|^2 - Mr \log \sigma_u^2 \\ &- \tfrac{1}{2\sigma_v^2} \sum_j \| v_j \|^2 - Nr \log \sigma_v^2\end{aligned}$$

How to determine Θ?

– Maximum likelihood estimate

$$\arg\max_\Theta \Pr(\mathbf{y} \mid \Theta) = \arg\max_\Theta \int \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta)\, d\mathbf{u}\, d\mathbf{v}$$

– Use the EM algorithm

Model Fitting: EM Algorithm

- Find

$$\hat{\Theta} = \arg\max_{\Theta} \Pr(\mathbf{y} \mid \Theta) = \arg\max_{\Theta} \int \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta)\, d\mathbf{u}\, d\mathbf{v}$$

- Iterate between E-step and M-step until convergence
  - Let $\hat{\Theta}^{(n)}$ be the current estimate
  - E-step: Compute $f(\Theta) = E_{(\mathbf{u},\mathbf{v} \mid \mathbf{y}, \hat{\Theta}^{(n)})}[\log \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta)]$

$$-\tfrac{1}{2\sigma^2}\sum_{(i,j)} E[(y_{ij} - u_i' v_j)^2] - \tfrac{1}{2\sigma_u^2}\sum_i E\| u_i \|^2 - \tfrac{1}{2\sigma_v^2}\sum_j E\| v_j \|^2$$
$$- R\log\sigma^2 - Mr\log\sigma_u^2 - Nr\log\sigma_v^2$$

    - The expectation is not in closed form
    - We draw Gibbs samples and compute the Monte Carlo mean

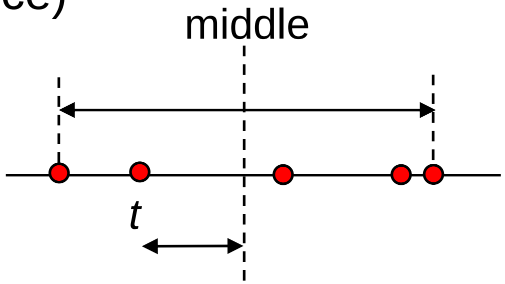
  - M-step: Find $\hat{\Theta}^{(n+1)} = \arg\max_{\Theta} f(\Theta)$

Example: timeSVD++

- Example of matrix factorization in practice
- Part of the winning method of Netflix contest [Koren 2009]

$$y_{ij,t} \sim \mu + \underbrace{b_i(t)}_{\text{user bias}} + \overbrace{b_j(t)}^{\text{item popularity}} + \underbrace{u_i(t)}_{\text{user factors (preference)}}' v_j$$

$$b_i(t) = b_i + \alpha_i \underbrace{\mathrm{dev}_i(t)}_{\text{distance to the middle rating time of } i} + b_{it}$$



$$b_j(t) = b_j + b_{j,\underbrace{\mathrm{bin}(t)}_{\text{time bin}}}$$

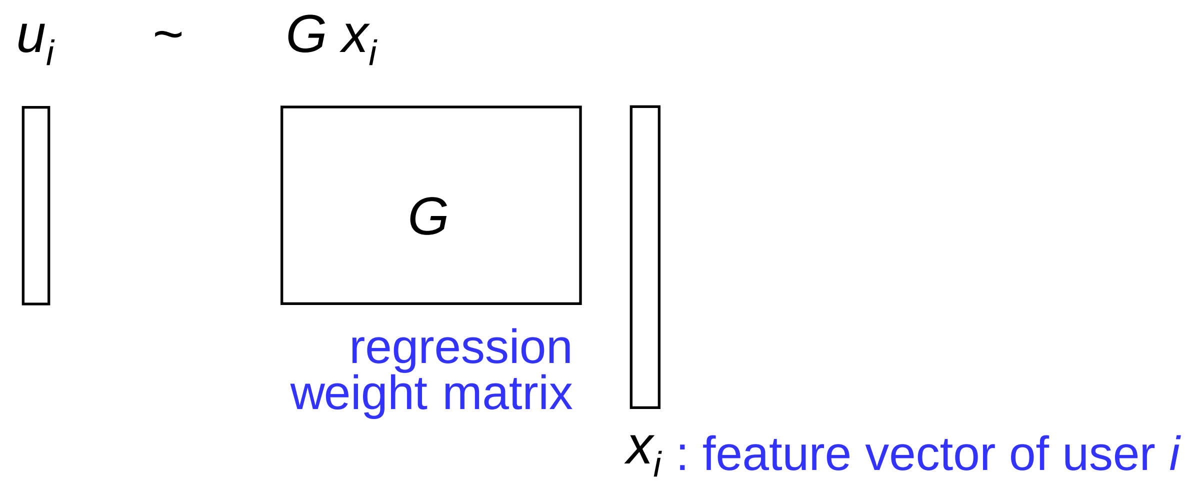
$$u_i(t)_k = u_{ik} + \alpha_{ik}\,\mathrm{dev}_u(t) + u_{ikt}$$

Model parameters: μ, b_i, α_i, b_{it}, b_j, b_{jd}, u_{ik}, α_{ik}, u_{ikt},
for all user *i*, item *j*, factor *k*, time *t*, time bin *d*

Subscript:
user *i*,
item *j*
time *t*

How to Handle Cold Start?

- For new items and new users, their factor values are all 0
- Simple idea
  - Predict their factor values based on features
    - For new user i, predict u_i based on x_i (user feature vector)



  - An item may be represented by a bag of words (later)

RLFM: Regression-based Latent Factor Model

- Incorporate features into matrix factorization
  - x_i: feature vector of user *i*
  - x_j: feature vector of item *j*
- Probabilistic model

$$y_{ij} \sim N(u_i' v_j,\ \sigma^2)$$

$$u_i \sim N(Gx_i,\ \sigma_u^2 I)$$

$$v_j \sim N(Dx_j,\ \sigma_v^2 I)$$

Let $\Theta = (G, D, \sigma^2, \sigma_u^2, \sigma_v^2)$

$$\begin{aligned}\log \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta) = \text{constant} \\ - \tfrac{1}{2\sigma^2} \textstyle\sum_{(i,j)} (y_{ij} - u_i' v_j)^2 - R \log \sigma^2 \\ - \tfrac{1}{2\sigma_u^2} \textstyle\sum_i \| u_i - Gx_i \|^2 - Mr \log \sigma_u^2 \\ - \tfrac{1}{2\sigma_v^2} \textstyle\sum_j \| v_j - Dx_j \|^2 - Nr \log \sigma_v^2\end{aligned}$$

Find

$$\hat{\Theta} = \arg\max_{\Theta} \Pr(\mathbf{y} \mid \Theta) = \arg\max_{\Theta} \int \Pr(\mathbf{y}, \mathbf{u}, \mathbf{v} \mid \Theta)\, d\mathbf{u}\, d\mathbf{v}$$

Comparison

- Zero-mean factorization

$$y_{ij} \sim N(u_i' v_j,\ \sigma^2)$$

$$u_i \sim N(0,\ \sigma_u^2 I)$$

$$v_j \sim N(0,\ \sigma_v^2 I)$$

- Factorization with features (RLFM)

$$y_{ij} \sim N(u_i' v_j,\ \sigma^2)$$

$$u_i \sim N(Gx_i,\ \sigma_u^2 I)$$

$$v_j \sim N(Dx_j,\ \sigma_v^2 I)$$

$$y_{ij} \sim N(x_i' G' D x_j + \delta_i' D x_j + x_i' G' \eta_j + \delta_i' \eta_j,\ \sigma^2)$$

$$u_i = Gx_i + \delta_i, \quad \delta_i \sim N(0,\ \sigma_u^2 I)$$

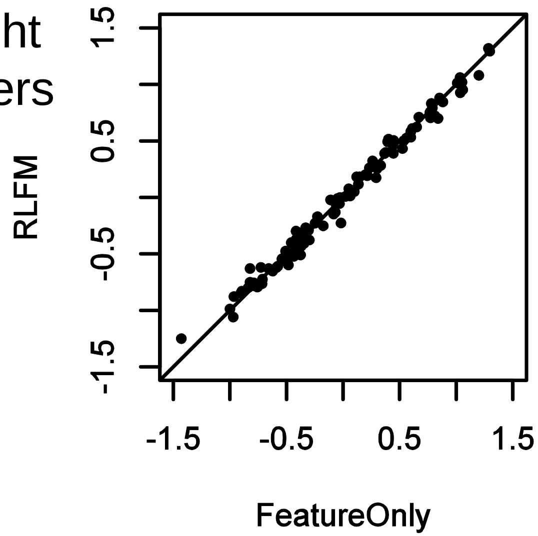
$$v_j = Dx_j + \eta_j, \ \eta_j \sim N(0,\ \sigma_v^2 I)$$

- Feature-only model

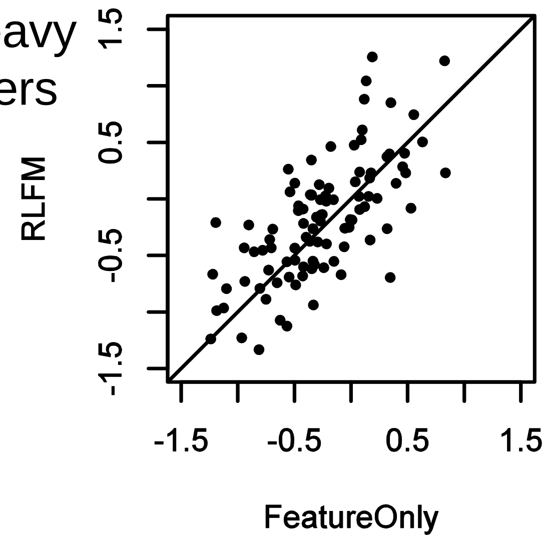
$$y_{ij} \sim N(x_i' G' D x_j,\ \sigma^2)$$

Illustration

Light users

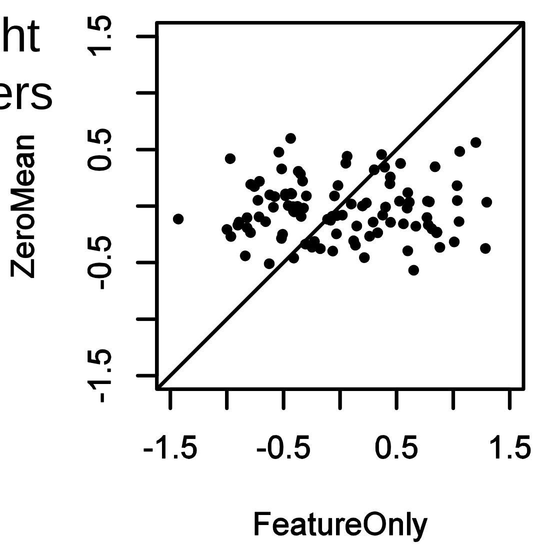


Heavy users

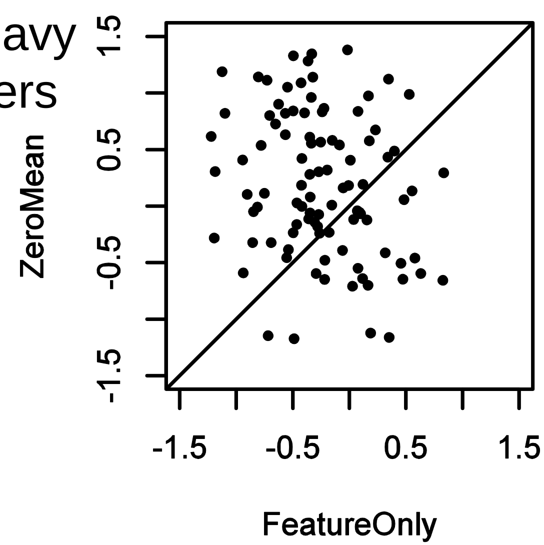


Factorization with features

Light users



Heavy users



Factorization without feature

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Non-linear RLFM

rating that user i gives item j

$$y_{ij} \sim b(x'_{ij}) + \alpha_i + \beta_j + u'_i\, v_j$$

x_i = feature vector of user i
x_j = feature vector of item j
x_{ij} = feature vector of (i, j)

- **Bias of user *i*:** $\alpha_i = g(x_i) + \varepsilon_i^{\alpha}, \quad \varepsilon_i^{\alpha} \sim N(0, \sigma_{\alpha}^2)$
- **Popularity of item *j*:** $\beta_j = d(x_j) + \varepsilon_j^{\beta}, \quad \varepsilon_j^{\beta} \sim N(0, \sigma_{\beta}^2)$
- **Factors of user *i*:** $u_i = G(x_i) + \varepsilon_i^{u}, \quad \varepsilon_i^{u} \sim N(0, \sigma_u^2 I)$
- **Factors of item *j*:** $v_i = D(x_j) + \varepsilon_i^{v}, \quad \varepsilon_i^{v} \sim N(0, \sigma_v^2 I)$

b, *g*, *d*, *G*, *D* are regression functions

Any regression model can be used here!!

fLDA: Factorization through LDA Topic Model

- An item is represented by a bag of word
- Model the rating y_{ij} that user *i* gives to item *j* as the user's affinity to the topics that the item has

User *i* 's affinity to topic *k*

$$y_{ij} = \ldots + \sum_k s_{ik} \bar{z}_{jk}$$

Pr(item *j* has topic *k*) estimated by averaging the LDA topic of each word in item *j*

The topic distribution z_{jk} of a new item *i* is predicted based on the bag of words in the item

  – Unlike regular unsupervised LDA topic modeling, here the LDA topics are learnt in a supervised manner based on past rating data
  – These supervised topics are likely to be more useful for the prediction purpose

Supervised Topic Assignment

The topic of the *n*th word in item *j*

Probability of observing y_{ij} given the model

$$\Pr(z_{jn} = k \mid \text{Rest})$$

$$\propto \underbrace{\frac{Z_{kl}^{\neg jn} + \eta}{Z_{k}^{\neg jn} + W\eta}\left(Z_{jk}^{\neg jn} + \lambda\right)} \cdot \underbrace{\prod_{i \text{ rated } j} \overbrace{f(y_{ij} \mid z_{jn} = k)}}$$

Same as unsupervised LDA

Likelihood of observed ratings by users who rated item *j* when z_{jn} is set to topic *k*

Experimental Results (MovieLens)

- Task: Predict the rating that a user would give a movie
- Training/test split:
  - Sort observations by time
  - First 75% → Training data
  - Last 25% → Test data
- User cold-start scenario
  - 56% test data with new users
  - 2% new items in test data

Model	Test RMSE
RLFM	0.9363
fLDA	0.9381
Factor-Only	0.9422
FilterBot	0.9517
unsup-LDA	0.9520
MostPopular	0.9726
Feature-Only	1.0906
Constant	1.1190

Summary

- Factorization methods usually have better performance than pure feature-based methods
  - Netflix
  - Our experience
- Metadata (feature vector or bag of words) can be easily incorporated into matrix factorization
- Next step
  - Matrix factorization with social networks
    - Friendship: Address book
    - Communication: Instant messages, emails
  - Multi-application factorization
    - E.g., joint factorization of the (user, news article) matrix and the (user, query) matrix

Fast Online Learning for Time-sensitive Recommendation

- Examples of time-sensitive items
  - News stories, trending queries, tweets, updates, events …
- Real-time data pipeline that continuously collects new ratings (clicks) on new items
- Modeling requirements:
  - **Fast learning**: Learn good models for new items using **little data**
    - Good initial guess (without ratings on new items)
    - Fast convergence
  - **Fast computation**: Build good models using **little time**
    - Efficient
    - Scalable
    - Parallelizable

FOBFM: Fast Online Bilinear Factor Model

Per-item online model $y_{ij} \sim u_i' \beta_j, \quad \beta_j \sim N(\mu_j, \Sigma)$

Subscript:
user i
item j
Data:
y_{ij} = rating that user i gives item j
u_i = offline factor vector of user i
x_j = feature vector of item j

- Feature-based model initialization

$$\beta_j \sim N(\underbrace{Ax_j}_{\text{predicted by features}}, \Sigma) \quad \Leftrightarrow \quad y_{ij} \sim u_i' A x_j + u_i' v_j$$

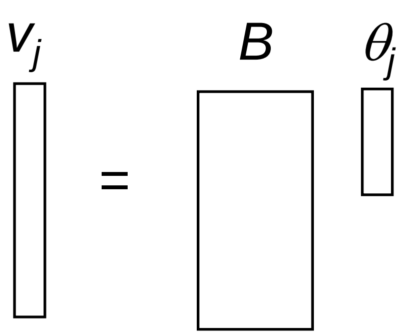
$$v_j \sim N(0, \Sigma)$$

- Dimensionality reduction for fast model convergence

$$v_j = B\theta_j$$

$$\theta_j \sim N(0, \; \sigma_\theta^2 I)$$

B is a $n \times k$ linear projection matrix ($k << n$)
project: high dim(v_j) $\rightarrow$ low dim(θ_j)
low-rank approx of Var[β_j]: $\beta_j \sim N(Ax_j, \; \sigma_\theta^2 BB')$



Offline training: Determine A, B, σ_θ^2 (once per day)

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- Fast, parallel online learning

$$y_{ij} \sim \underbrace{u_i' A x_j}_{\text{offset}} + \underbrace{(u_i' B)}_{\text{new feature vector (low dimensional)}} \theta_j,$$

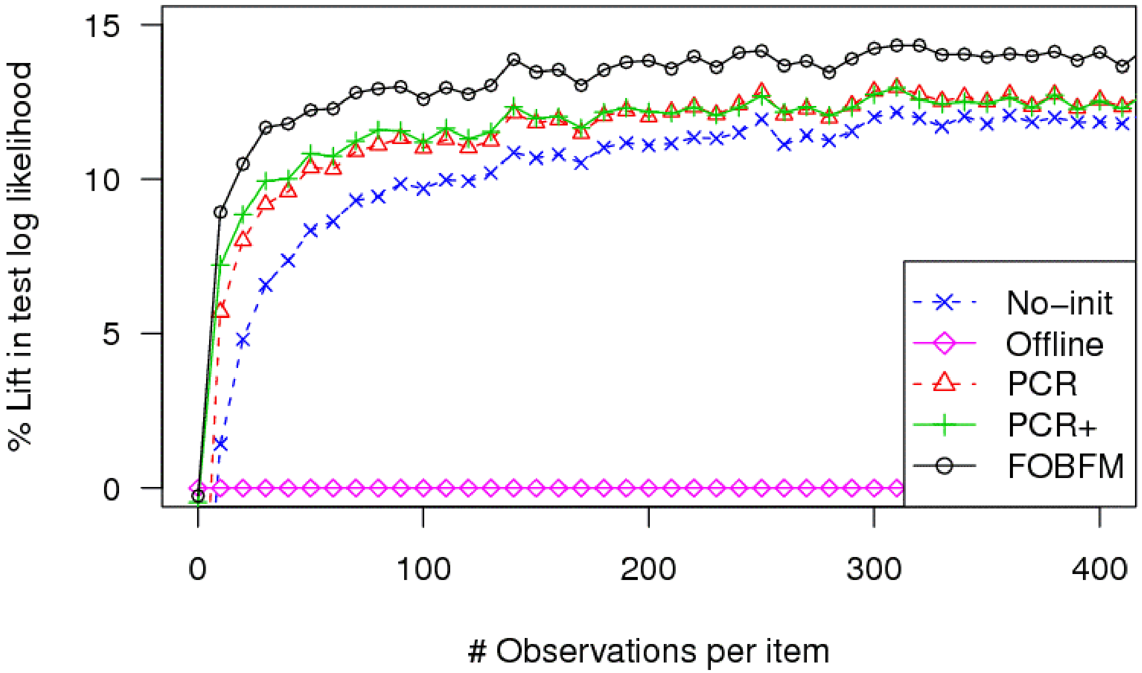
where θ_j is updated in an online manner

- Online selection of dimensionality (k = dim(θ_j))
  – Maintain an ensemble of models, one for each candidate dimensionality

Experimental Results: My Yahoo! Dataset (1)

- My Yahoo! is a personalized news reading site
  - Users manually select news/RSS feeds
- ~12M “ratings” from ~3M users to ~13K articles
  - Click = positive
  - View without click = negative

Experimental Results: My Yahoo! Dataset (2)

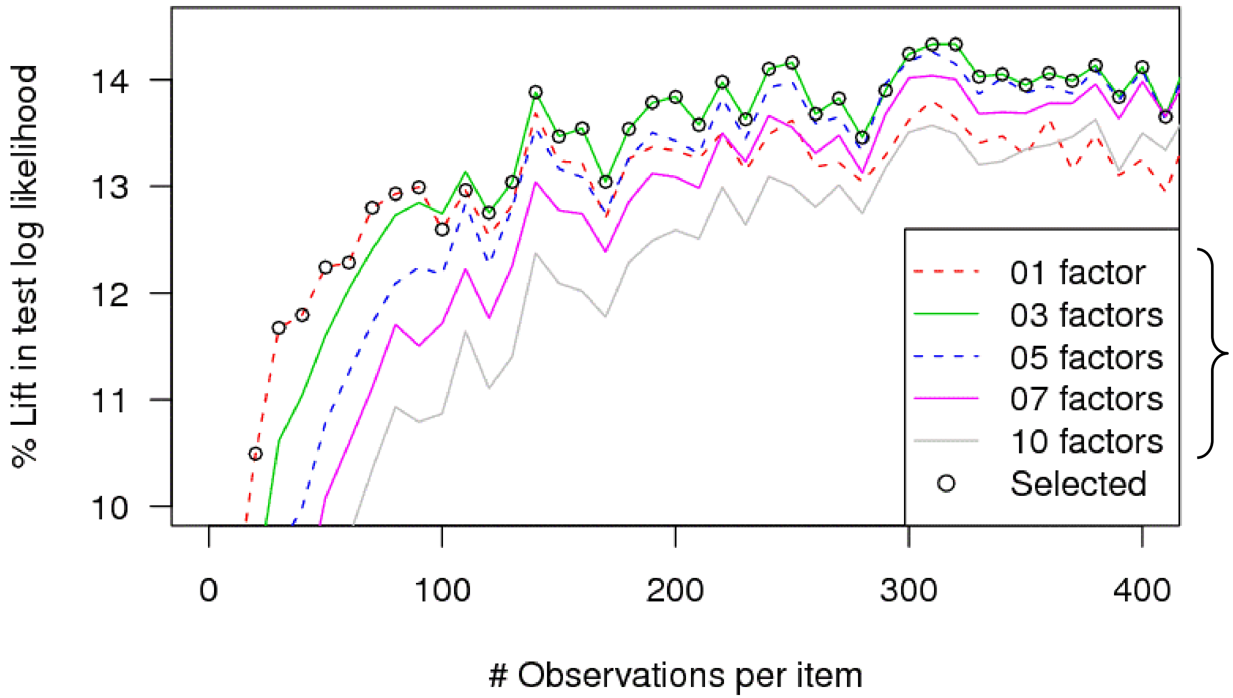


Methods:

- **No-init:** Standard online regression with ~1000 parameters for each item
- **Offline:** Feature-based model without online update
- **PCR, PCR+:** Two principal component methods to estimate *B*
- **FOBFM:** Our fast online method

- Item-based data split: Every item is new in the test data
  - First 8K articles are in the training data (offline training)
  - Remaining articles are in the test data (online prediction & learning)
- Our supervised dimensionality reduction (reduced rank regression) significantly outperforms other methods

Experimental Results: My Yahoo! Dataset (3)



factors = Number of parameters per item updated online

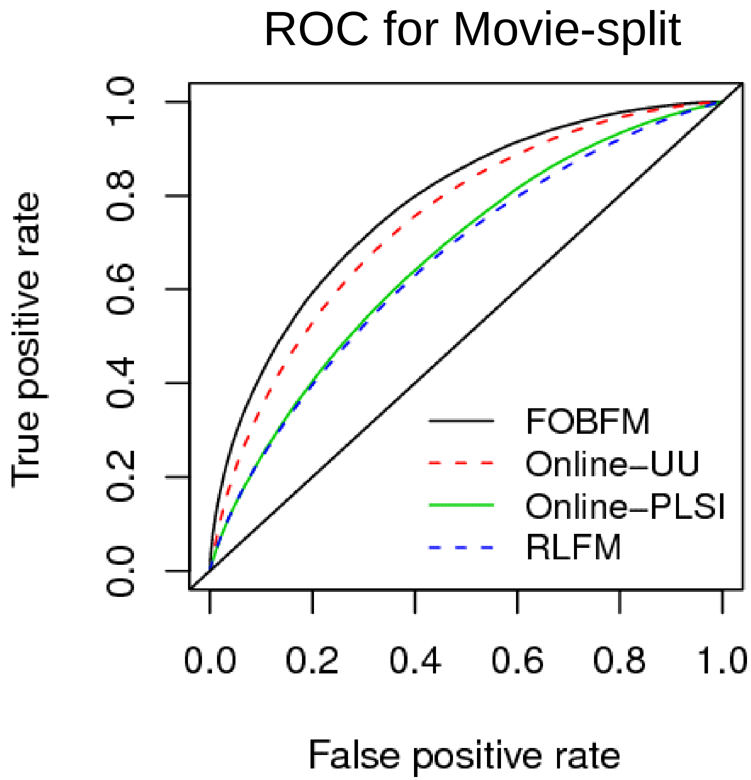
- Small number of factors (low dimensionality) is better when the amount of data for online leaning is small
- Large number of factors is better when the data for learning becomes large
- The online selection method usually selects the best dimensionality

Experimental Results: MovieLens Dataset

- Training-test data split
  - Time-split: First 75% ratings in training; rest in test
  - Movie-split: 75% randomly selected movies in training; rest in test

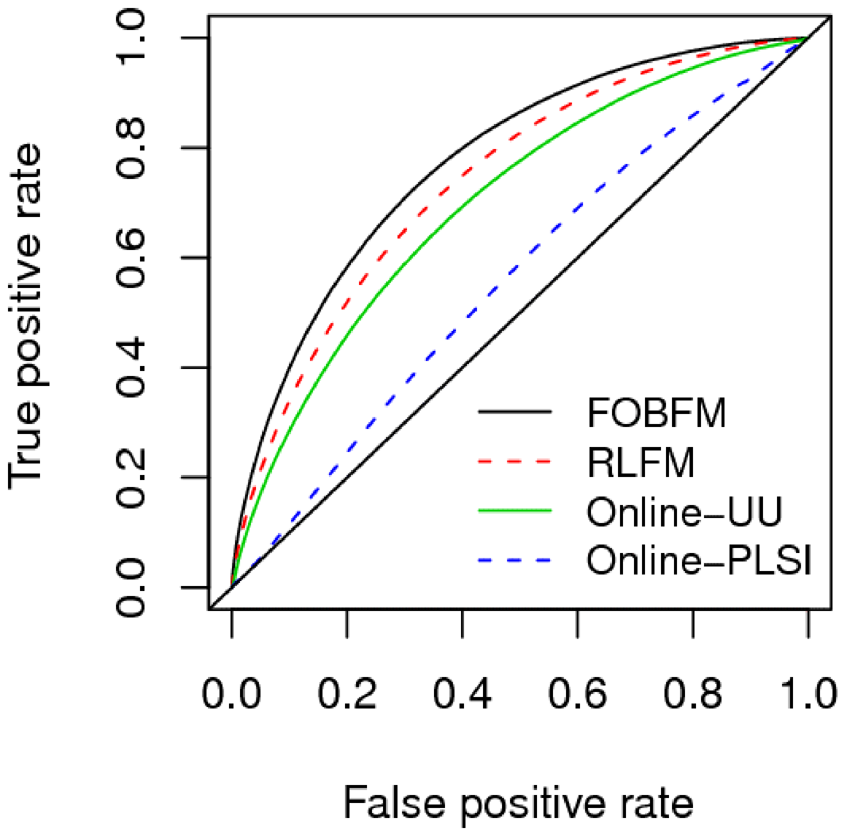
Model	RMSE Time-split	RMSE Movie-split
FOBFM	0.8429	0.8549
RLFM	0.9363	1.0858
Online-UU	1.0806	0.9453
Constant	1.1190	1.1162

FOBFM: Our fast online method
RLFM: [Agarwal 2009]
Online-UU: Online version of user-user collaborative filtering
Online-PLSI: [Das 2007]



Experimental Results: Yahoo! Front Page Dataset

- Training-test data split
  - Time-split: First 75% ratings in training; rest in test



–~2M “ratings” from ~30K frequent users to ~4K articles

•Click = positive

•View without click = negative

–Our fast learning method outperforms others

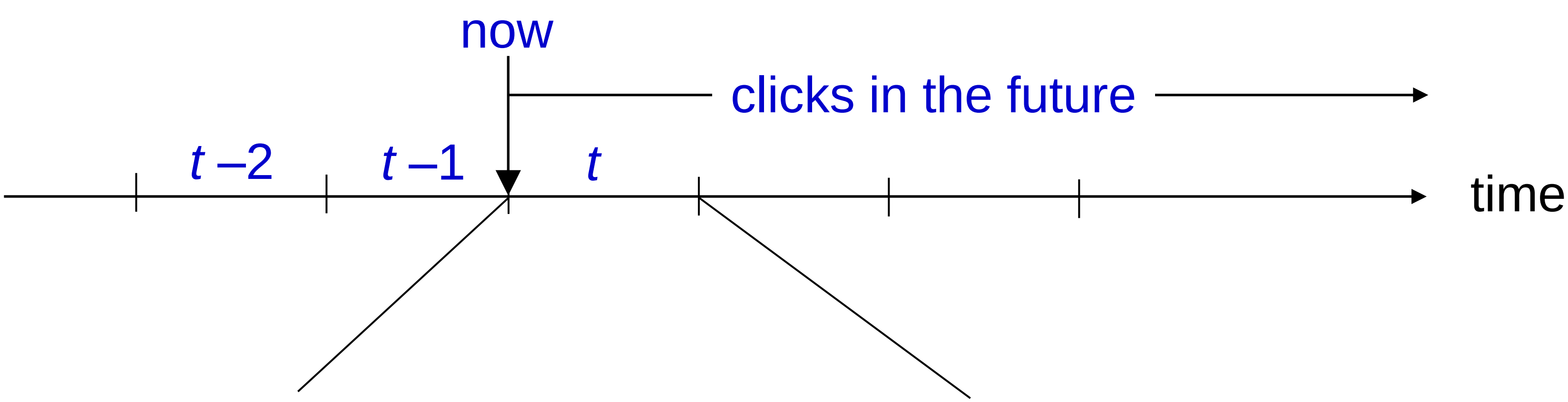
Summary

- Recommending time-sensitive items is challenging
  - Most collaborative filtering methods do not work well in cold start
  - Rebuilding models can incur too much latency when the numbers of items and users are large
- Our approach:
  - Periodically rebuild the offline model that
    - uses feature-based regression to **predict the initial point** for online learning, and
    - **reduces the dimensionality** of online learning
  - Rapidly update online models once new data is received
    - Fast learning: Low dimensional and easily parallelizable
    - Online selection for the best dimensionality

Important Problems Beyond Factor Models

- How to explore/exploit with small traffic, a large item pool, at a fine granularity
- Offline evaluation
- Multi-objective optimization under uncertainty
- Whole page optimization

Explore/Exploit



Show Item 1 with probability x_1
Item 2 x_2
…
Item K x_K

ICDM'09 (best paper)
- Small number of items
- No deep personalization

Determine (x_1, x_2, …, x_K) based on clicks and views observed before t in order to maximize the expected total number of clicks in the future

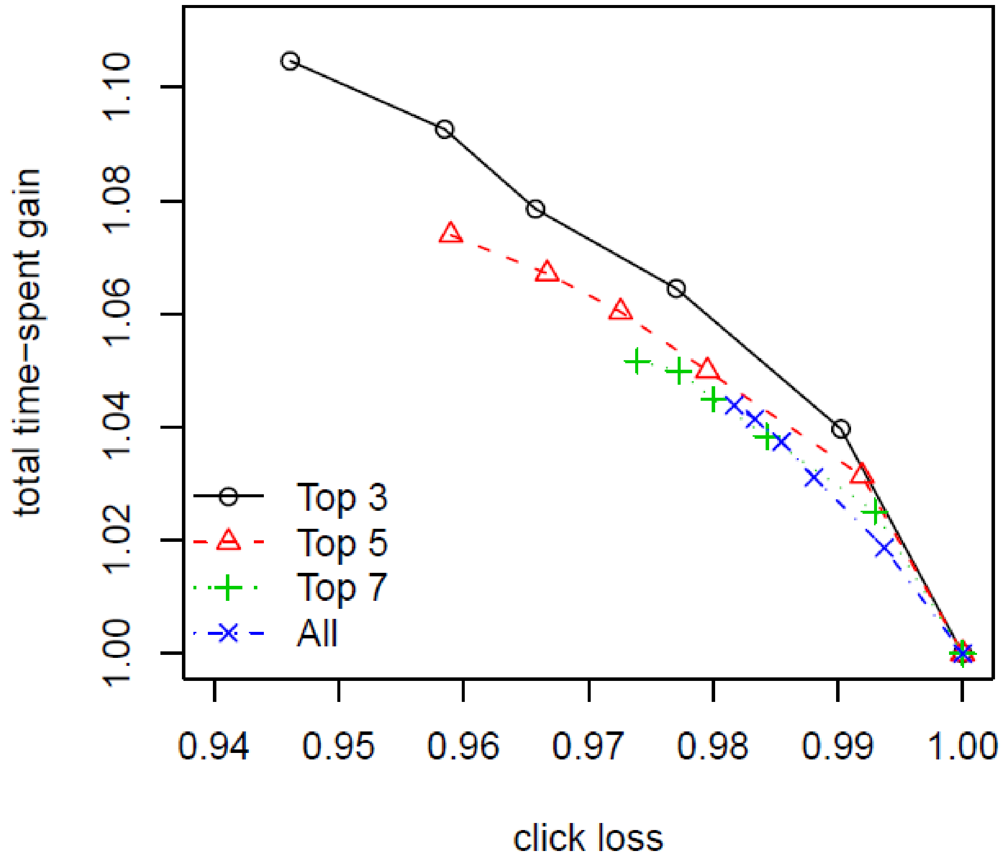
Challenges
- **Large number of items**
- **Small traffic**
- **Deep personalization**

Offline Evaluation

- Ultimate evaluation: Online bucket test
- Unbiased offline evaluation based on random-bucket data
  - [Lihong Li, WWW'10, WSDM'11]
  - Random bucket: A small user population to which we show each item with equal probability
  - Assumptions:
    - Single recommendation per visit (instead of top-*K*)
    - All the users respond to the recommended item in an *iid* manner
  - Replay-match methodology
- Challenges
  - How to handle non-random data
  - How to extend to top-*K* recommendation
  - How to capture users' "non-*iid*" behavior in a session

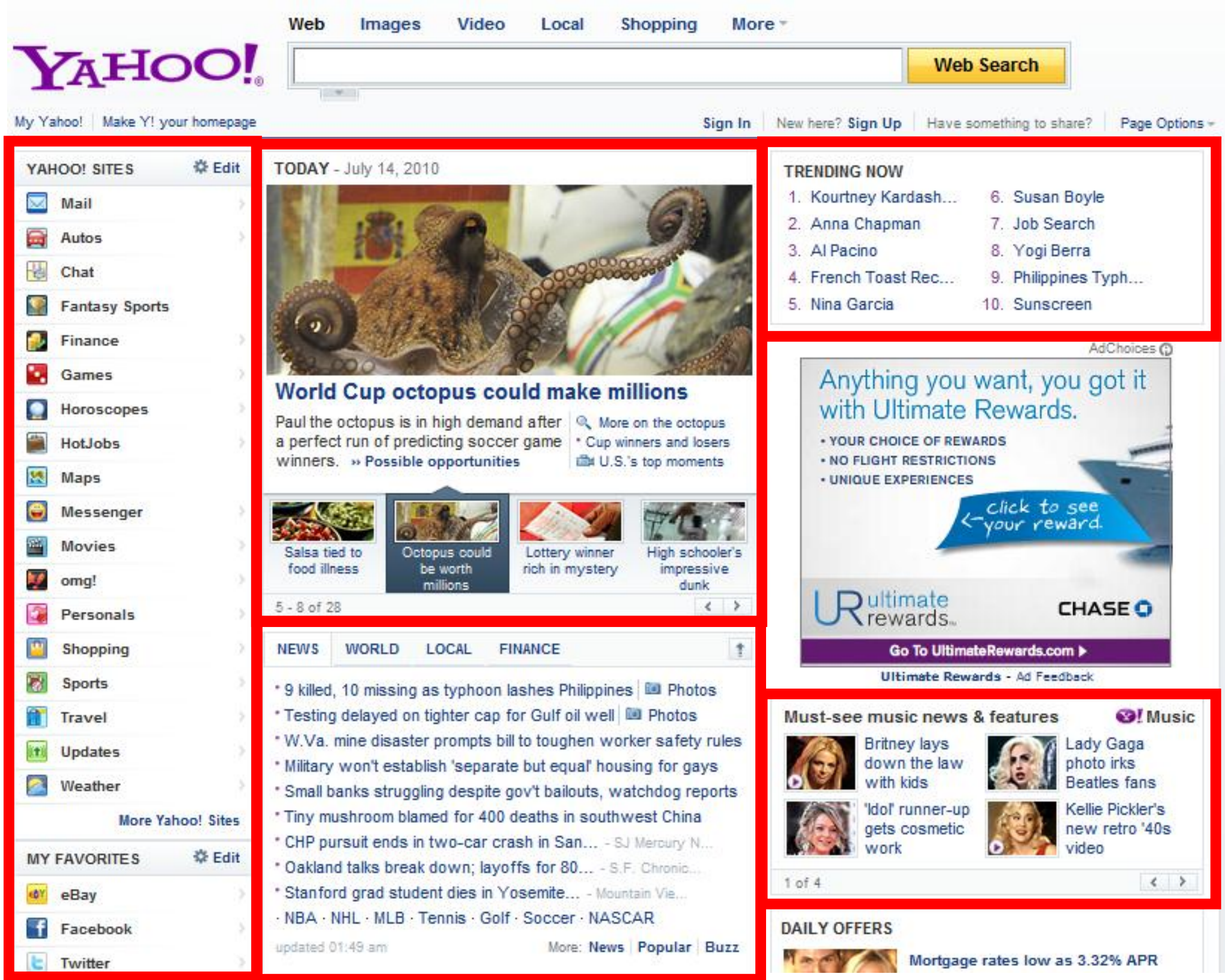
Multi-Objective Optimization

- Maximize time-spent (or revenue) s.t. click drop < 5%



Challenges:
- Deep personalization
- Optimization in the presence of uncertainty

Whole Page Optimization



Challenge:
How to jointly optimize all these modules

- Diversity
- Consistency
- Relatedness

Thank You!

Contact me for job/internship opportunities in Yahoo! Labs

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